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ABSTRACT

The concept of persistency of river discharges has been introduced in a qualitative sense in 1960 at the Helsinki assembly.

The present paper illustrates this important feature with quantitative information. The influence of former discharges on actual discharges is investigated.

1. INTRODUCTION

In the paper, presented by the author at the Helsinki congress 1960 (*) about the same subject, special attention has been paid to a characteristic feature of riverdischarges, named persistency. Discharge, though caused by precipitation, does not show the same sequence of *H*- or plus values and *L*- or minus values (plus and minus in relation to the general mean value) as is demonstrated by the precipitation figures. Monthly mean precipitation values prove to be subject to a stochastic distribution. Monthly mean discharges however show long periods of *H*-values and long periods of *L*-values, which can not be explained by accident. There is not a stochastic distribution of independent values. On the contrary, the subsequent months are closely related one to another. There is a pronounced tendency of the discharge to persist in a once established state.

From the above study may be reproduced here fig. 1 giving the persistencies on the Rhine for the space of time 1919-1959. In this diagram the time runs along the *X*-axis. *H*-months are drawn upwards from the zero line, *L*-months downwards, to the total length of every period.

This study has been based exclusively on the sequence of *H*- or *L*-values. The values itself have been ignored. Only the probability law did give the decision. So diagram 1 shows only the duration of the persistencies and their manifold occurrences. In this second paper we will make use of the proper discharge values and present the persistencies in a quantitative sense.

It should have been possible to demonstrate the existence, as well as their general character in a quantitative sense more or less directly with the discharge figures themselves, For this purpose we could have used a simple graphical presentation of the monthly mean values or we could have constructed a graph with moving averages.

But this should not have added essential knowledge or better understanding. Moreover moving averages have their centre some months before the present one. They can never be up "to date", which is, if we wish to forecast, at least a disadvantage.

A more fundamental possibility to investigate the essential properties of persistencies one might expect to find in correlation theory. This is however a theory of statistical relations, more or less suitable to operate with before one has made acquaintance with the existence of persistencies. Once this existence being recognised a strong demand is arising to a causal theory of dependency, which in principle cannot be provided by correlation.

Such a causal theory will be presented here. It has some relationship with unit hydrograph theory. But there is at the same time an essential difference. Unit hydro-

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(*) International association of scientific hydrology, Publ. 51.

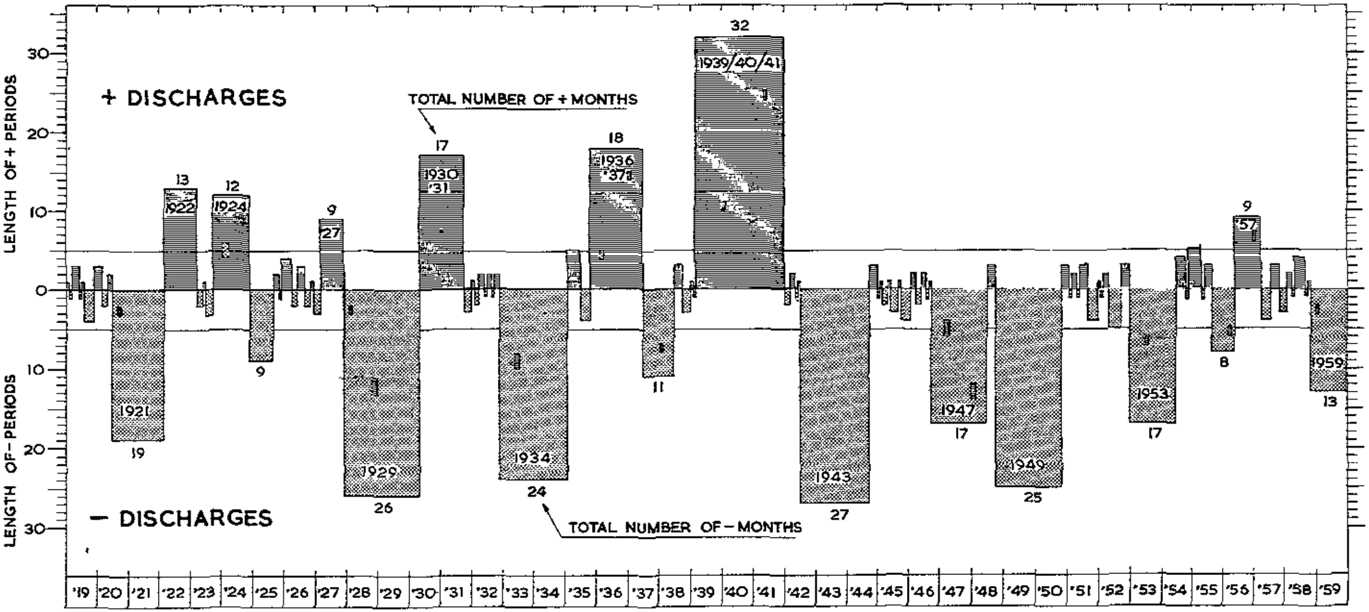

Fig. 1 — Persistencies in the runoff of the Rhine, as deduced from mean monthly values.

graph concerns the description of the single flood and the individual runoff hydrograph, It considers hours, days, perhaps weeks.

Our present theory deals only with monthly mean values and pays special attention to periods of many months. Though both theories will have a point of contact in some characteristics of the catchment I have confined this presentation to the aspects which are related to persistencies.

Perhaps it will be better not to speak of a theory, but of a model. The calculation as introduced here implies a special, and of course strongly simplified conception of the runoff mechanism of precipitation. This may be called a model, being more or less in correspondence with reality. But as far as concerned to persistencies, this model has proved to be of full use.

2. THE MECHANISM OF BASE RUNOFF

The scheme, developed to describe the causal relations between discharge and persistency, may be presented as a coherent set of algebraic formulae. We indicate :

A = monthly mean discharge of the river
G = groundwater runoff, resulting from precipitation in former months
G' = groundwater runoff as increased in the month itself by a part of the precipitation
R = increase in the actual month from "old" baseflow G to "new" baseflow G'
P = netto precipitation
V = basin storage from which baseflow G results
V' = basin storage from which baseflow G' results
D = replenishment of the basin storage V in month n to V'

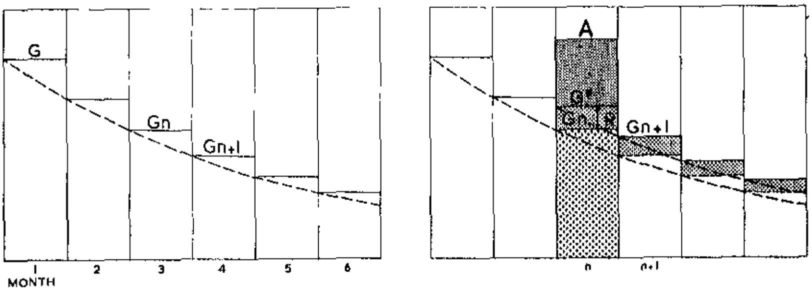

Fig. 2 — Base runoff without replenishment (left) and with replenishment in month n (right).

a) We start with two fundamental equations :

$$G_{n+1} = r \,.\, G_n \tag{1}$$

$$R = p \,.\, (A - G) \tag{2}$$

Proportionality between G_n and G_{n+1} is in accordance with Darcy's law. In this paper we will take the reduction factor r as constant. There seems however no objection to adapt r if conditions in time as well as in space should be varying.

The coefficient p is called storage factor. It is a well-known fact that a part of netto precipitation P gives direct runoff and that another part infiltrates and causes a delayed runoff or baseflow. In our calculation scheme we will start with an unknow value of P. If we take G for granted, then the direct runoff $(A-G)$ is known and with (2) we introduce the new starting point for the month $n+1$:

$$G' = G + R = G + p\,.\,(A - G) \tag{3}$$

The base runoff G in the next month $n+1$ is with (1) and (3) defined by

$$G_{n+1} = r\,.\,G'_n = r\,.\,[G + p\,.(A - G)] \tag{4}$$

It should be noted, that R is a part of the direct runoff $(A - G)$ in month n. It is however allowed to conceive R as a part of baseflow as well. The two conceptions are reconciled if one takes R as that part of precipitation that infiltrates but already in the month n comes to runoff. So G' may be seen as composed of two parts : first of G which is old baseflow. Second of R which is that part of new baseflow which comes to runoff in the proper month. This part can not be discerned from direct runoff and in this paper it is considered as direct runoff.

Equation (4) gives a key to calculate the time history of G from a given series of values of A, if only a starting value G for the first month is accepted. This scheme will be introduced in par. 4.

b) If there is a groundwater runoff G we must admit the existence of a basin storage V. Without any replenishment this storage must be sufficient for a series of runoff, as defined by (1), so

$$V_n = G_n(1 + r + r^2 + r^3 + \ldots) = G_n \frac{1}{1 - r} \tag{5}$$

$$V_R = \frac{p\,.(A - G)}{1 - r} \tag{6}$$

$$V' = V + V_R = \frac{1}{1 - r} G' = \frac{1}{1 - r} [G + p\,.(A - G)] \tag{7}$$

$$V_{n+1} = r\,.\,V' = \frac{r}{1 - r} [G + p\,.(A - G)] \tag{8}$$

$$D = V_{n+1} - V_n = \frac{p\,.\,r}{1 - r}(A - G) - G = r\,.\,V_R - G = P_g - G \tag{9}$$

c) Normally runoff is caused by netto precipitation, being precipitation minus evaporation and evapotranspiration, in this paper indicated as P. So P is that part of physical precipitation that comes to runoff in the river, partly as "direct runoff" P_d in the proper month, partly as delayed runoff P_g in following months. So

$$P_d = (A - G) \tag{10}$$

$$P_g = p\,.\,(A - G)\,(r + r^2 + r^3 + \ldots) = \frac{p\,.\,r}{1 - r}(A - G) = r\,.\,V_R \tag{11}$$

$$P = P_d + P_g = (A - G)\left[1 + \frac{p\,.\,r}{1 - r}\right] = A + D \tag{12}$$

d) It may be of value to indicate two special conditions of runoff. First the condition of constant equilibrium. *If P is constant, during many months the same* mean value P_c, then imperatively (with index $_c$ for constant)

$$P_c = A_c \tag{13}$$

which leads to the equilibrium relations

$$G_c = \frac{p.r}{1 - r + p.r} A_c = r.G'_c = (A_c - G_c)\,\frac{p.r}{1 - r} = P_{gc} \qquad (14)$$

$$V_c = \frac{p.r}{(1 - r)(1 - r + p.r)} A_c = V_{n+1/c} \qquad (15)$$

$$D_c = 0$$

If p and r are taken constant, the relations (13), (14) and (15) are applicable to the overall mean values of A, G, P etc. over a series of years.

e) As a second special condition may be considered the abrupt entrance of a period of total drought. This condition marked $''$ is characterised by

$P'' = 0$ and hence $\qquad (A'' - G'') = 0$ so
$A'' = G'' \qquad R'' = 0 \qquad G'^{\prime\prime} = G''$

It starts a period of base runoff as defined by (1) and as illustrated by fig. 2 left. So A never can be less than G. Otherwise, if one calculates G with the developed formulae, it never may surpass A, for this is physically meaningless. This gives a criterium for the determination of p and r as will be shown.

f) The coherence of the different quantities is demonstrated in fig. 3. This diagram shows the formulae (5), (8), (9), (10), (11) and (12) in their physical relations.

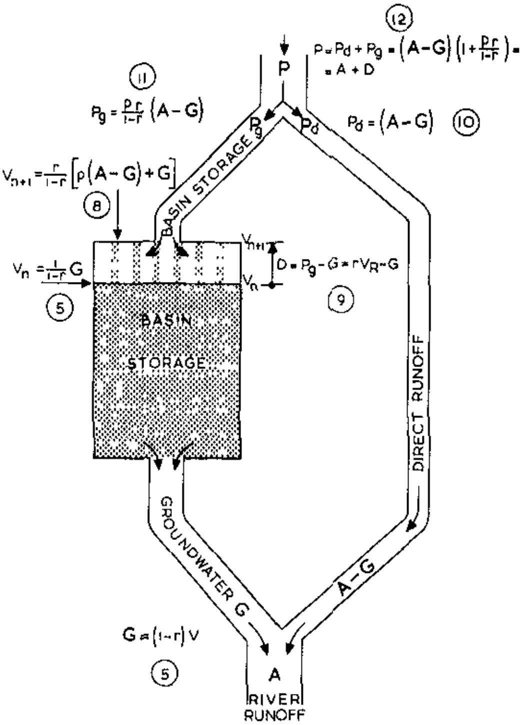

Fig. 3 — The coherence of the different relations, which determine the division of precipitation over direct runoff and base runoff and otherwise, the composition of the actual discharge by direct runoff and base runoff.

g) A special remark has to be devoted to the dimensions in which the formulae have been presented here. They have deliberately been left away, but of course, they have dimensions.

Choosing m^3/ sec for A, G, G' and R no further explanation is required, nor for P. As to V, V' and D one finds these quantities in m^3/sec. Finding e.g. $V = 1000$ m^3/sec it is obvious that the meaning of this is, that the groundwater storage is able to supply 1000 m^3/sec during one month, if one month has been established as unit of time. The next month base runoff will then be reduced to r. 1000 m^3/sec. So the total amount of groundwater, as defined by (5) is found if one multiplies $1000/(1-r)$ with $2{,}63.10^6$, being the quantity of seconds in one month.

Though time plays an essential role in the dynamic process of runoff, as meant here, it does not enter into our formulae in an explicit form. Our formulae have been built up for units of time as e.g. one month and give the relations between month n and month $n+1$. In stead of a month it is of course admitted to use a fortnight or a two-month period as unit. The true length of the choosen period comes however fully to action in the coefficients. If r should be 0,8 for a one-month time unit it is 0,64 for a two-month unit and 0,51 for a three-month unit of time. And p being e.g. 0,4 for a one-month unit of time it is 0,2 for a half-month unit of time.

3. THE COEFFICIENTS p AND r

The tools of the developed model are confined to the coefficients p and r. So this method to describe the dynamical relation between direct runoff and base runoff may be mentioned PR-model.

It will be obvious, that it is impossible to derive the reduction factor r and the storage factor p directly from field-informations. We dispose however over information in relation to the action and influence of p and r, if we look at the general behaviour of a river. So measuring the monthly mean values of runoff one may say to have measured indirectly the coefficients p and r, which will be demonstrated in par. 4.

These factors will be different for various rivers on account of differences in slope, in soil conditions, vegetation etc. They will even be different for tributaries and separate areas of the catchment of one large river. It may also be assumed, that these two coefficients will not be independent of season, meteorological conditions, preceeding situations etc. So even for a certain river it may be expected that p and r will demonstrate some variability in time as well as in space.

This variability however may be put aside for further investigations. The results we obtained with practically constant values for p and r are so striking and valuable, that they may be presented here, without paying further attention to their potential variability.

Of course p and r must lie between 0 and 1. The values $p = 0$ and $r = 0$ indicate a river basin of impermeable rock, without any infiltration. It will be evident, that very small values of p and r belong to a river with very small retention capacity where persistencies cannot exist at all.

The value $p = 1$ indicates a catchment, acting as a sponge absorbing precipitation for 100%. In this case r indicates the tenacity of this sponge. The value $r = 1$ should mean that all precipitation is held in the basin for all time.

A noticeable relationship with persistencies must be expected for values of p and r which differ sufficiently from zero. And the existence of persistencies may be regarded as a *strong indication*, that fairly large values for p and r will be found.

4. A CALCULATION SAMPLE

By a way of trial and error several values of p and r have been tried out. It proved to be not very difficult to conclude to coherent values which tentatively enables to execute the developed scheme of calculation.

In order to elucidate the existence and the character of the persistencies, the calculation has been executed for the river Rhine. Its catchment area is 165.000 km^2. We used the discharge figures for the gauge Lobith, situated at 160 km from the mouth of that river. Mean discharge 2200 m^3/sec, minimum over a few days 600 m^3/sec, minimum monthly mean discharge 680 m^3/sec, maximum monthly mean discharge 7350 m^3/sec, maximum measured discharge 13.000 m^3/sec.

For our investigation we could make use of monthly mean discharge values over a period of 62 years, being 1901-1962.The reduction coefficient r has been taken constant at 0,8 for the whole period, all seasons, and disregarding actual runoff or the runoff figures of preceeding months. As already mentioned in par. 3 differences in the character of the tributaries and of geological provinces have not been taken into account. So this constant factor 0,8 must be regarded as an overall value for the whole catchment.

The storage factor p has been taken constant for a wide range of months (85%) at the value 1/3. For values of A greater than 3000 m^3/sec up to the maximum of 7350 m^3/sec a reduction has been applied, being respectively for each range of 1000 m^3/sec above 3000 m^3/sec 1/3,5 1/4 1/4,5 1/5 1/5,5 1/6. The physical justification of this reduction is, that from greater amounts of precipitation a smaller part infiltrates.

The criteria after which p and r could be determined have been found in the necessity to avoid negative values for $A - G$, which of course are physically valueless. The second criterium was found in the similarity which necessarily must exist between the general trend of the At-curves and the trend of the Gt-curve in a sequence of falling months.

Admittedly this procedure is not free from subjective influence. Possibly in future there will be found a more exact manner to determine p and r from monthly mean discharge values or any other way. But the results obtained are convincing enough to justify their presentation.

Moreover differences in the character of the tributaries do lead now and then to deviations which can hardly be put aside without a more thorough investigation. So we propose r as being in our special investigation constant 0,8 and p as nearly constant 1/3, both values only as a first attempt to describe the mechanism of persistencies. Table 1 is giving a sample of the calculation.

The period chosen, column 1, is arbitrary. Column 2 gives the monthly mean discharges in m^3/sec, as measured in Lobith in the months in question. The values of p are given in column 5, corresponding to the system, uniformly used for the whole calculation. For r is uniformly used the value 0,8.

With the values of A and the coefficients p and r we are able to start the calculation by guessing the first value of G, in our sample 1120. Starting with $A = 1220$ and $G = 1120$ the calculation runs horizontally through $(A - G) = 100$, with $p = 1/3$ we find $p(A - G) = 30$, $G' = 1150$, $G_{n+1} = 920$. This last value is the starting value of G in the next month, a.s.o.

This calculation scheme has a most noteworthy feature. In every line G represents the total of all past influences as will be illustrated in par. 9. But the sum of these influences is continuously reduced, partly by p, partly by r. The coefficient r reduces every deviation with 20% per month, so in the course of 10 months to 0,80, 0,64, 0,51, 0,41, 0,33, 0,26, 0,21, 0,17, 0,13, 0,11 times the original deviation.

The result is that the influence of very large or very small values of A is still great on the month immediately following, but is reduced and gradually runs to zero for months coming much later. This will be discussed in following paragraphs.

TABLE 1

Sample of calculation of groundwater runoff only from monthly mean discharge figures. All values are expressed in m^3/sec

month	A	G	$A-G$	p	$p\,(A-G) = R$ (2)	$G+R = G'$ (3)	$rG' = G_{n+1}$ (4)	$rG'/1-r = V_{n+1}$ (7), (8)	D: + = repl.; − = loss (9)	$A+D=P$ (12)
1	2	3	4	5	6	7	8	9	10	11
’61 O	1220	1120	100	1/3	30	1150	920	4600	−1000	220
N	1560	920	640	1/3	210	1130	900	4520	− 80	1480
D	2900	900	2000	1/3	670	1570	1260	6280	+1760	4660
’62 J	2910	1260	1650	1/3	550	1810	1450	7240	+ 960	3870
F	3780	1450	2330	1/3,5	660	2110	1690	8440	+1200	4980
M	2290	1690	600	1/3	200	1890	1510	7560	− 880	1410
A	3990	1510	2480	1/3,5	710	2220	1780	8880	+1320	5310
M	2570	1780	790	1/3	260	2040	1630	8160	− 720	1850
J	2130	1630	500	1/3	170	1800	1440	7200	− 960	1170
J	1740	1440	300	1/3	100	1540	1230	6160	−1040	700
A	1470	1230	240	1/3	80	1310	1050	5240	− 920	550
S	1210	1050	160	1/3	50	1100	880	4400	− 840	370
O	940	880	60	1/3	20	900	720	3600	− 800	140
N	840	720	120	1/3	40	760	610	3040	− 560	280
D	1330	610	720	1/3	240	850	680	3400	+ 360	1690

With $r = 0{,}8$ the fraction $r/(1 - r) = 4$. So one finds the total groundwater storage V_{n+1}, column 9, by multiplication column 7 with 4. The monthly replenishment or loss of storage is easily found with (9) : see column 10. The monthly netto precipitation is found with (12) to be as produced in column 11.

It lies byond the scope of this paper to compare these precipitation figures, calculated from the discharge figures, with those, obtained from meteorological sources. It may suffice here to indicate, that the PR-calculation introduces a method to arrive at a new dynamic approach to the relation between precipitation and runoff. The short sample illustrates that the "calculated" precipitations N differ greatly from the runoff figures A. Yet one can easily ascertain, that over a sufficiently long period of time $A = A + D = P$. So it is a characteristic feature of the PR-calculation, that it provides a new distribution in time for P, but it is not the purpose of this paper to call attention to this possibly important feature.

5. BASIN STORAGE CURVE

The main purpose of the present investigation is to find causal relations which can explain fundamentally as well as quantitavely the persistencies. Having supposed already in the Helsinki paper that the storage capacity of the catchment might be responsible we will now pay attention to the quantity V_{n+1}. After (8) this is equal to $r.\ V'$, being the basin storage which causes base runoff in the next and following months.

Fig. 4 demonstrates what has been found. Merely to avoid reproduction difficulties the diagram has been confined to the space of time as indicated, which coincides with the space of time of fig. 1.

The calculation sample of table 1 is a part of this diagram and may serve as an illustration vice versa. As to the translation of the ordinates of this graph into m^3 for the whole catchment may be referred to par. 10.

The main character of fig. 4 is the fluctuating course of this graph round the general mean value, being for the full 62-year period about 6000 m^3/sec.

6. IDENTITY BETWEEN STORAGE CURVE FLUCTUATIONS AND PERSISTENCIES

In order to indicate the causal relation between the persistencies and the groundwater storage, we have to compare the dates and durations as shown by fig. 1 with the dates and durations (of course not the amplitudes) as shown by fig. 4. Some unimportant differences are due to the fact, that fig. 1 is based on the difference of every monthly mean value with the overall mean value for that month, while fig. 4 is based on comparison of V_{n+1} of every month with the general mean value 6000 m^3/sec.

Fig. 1 shows 7 H-persistencies and 10 L-persistencies, if we neglect 1959, not having ended in December 1959. To start with the L-persistencies the mean duration of these 10 persistencies is 18 months. In fig. 4 we encounter the corresponding 10 periods of low V_{n+1} values beginning and ending at almost exactly the same months. In our calculation the duration of these 10 persistencies is 18 months as well.

Moreover fig. 4 presents 2 more L-persistencies being 7 months (1945) and 9 months (1957).

As to the H-persistencies fig. 1 shows a number of 7 with a mean duration of 16 months. Fig. 4 shows on the corresponding periods as well 7 persistencies, but now with a mean duration of 20 months. Moreover there are in fig. 4 another 9 persistencies in the same space of time with a mean value of 8 months.

So the present investigation leads to 28 persistencies against 17 in the former one and a total of months in condition of persistency of 409 against 293 in the former one.

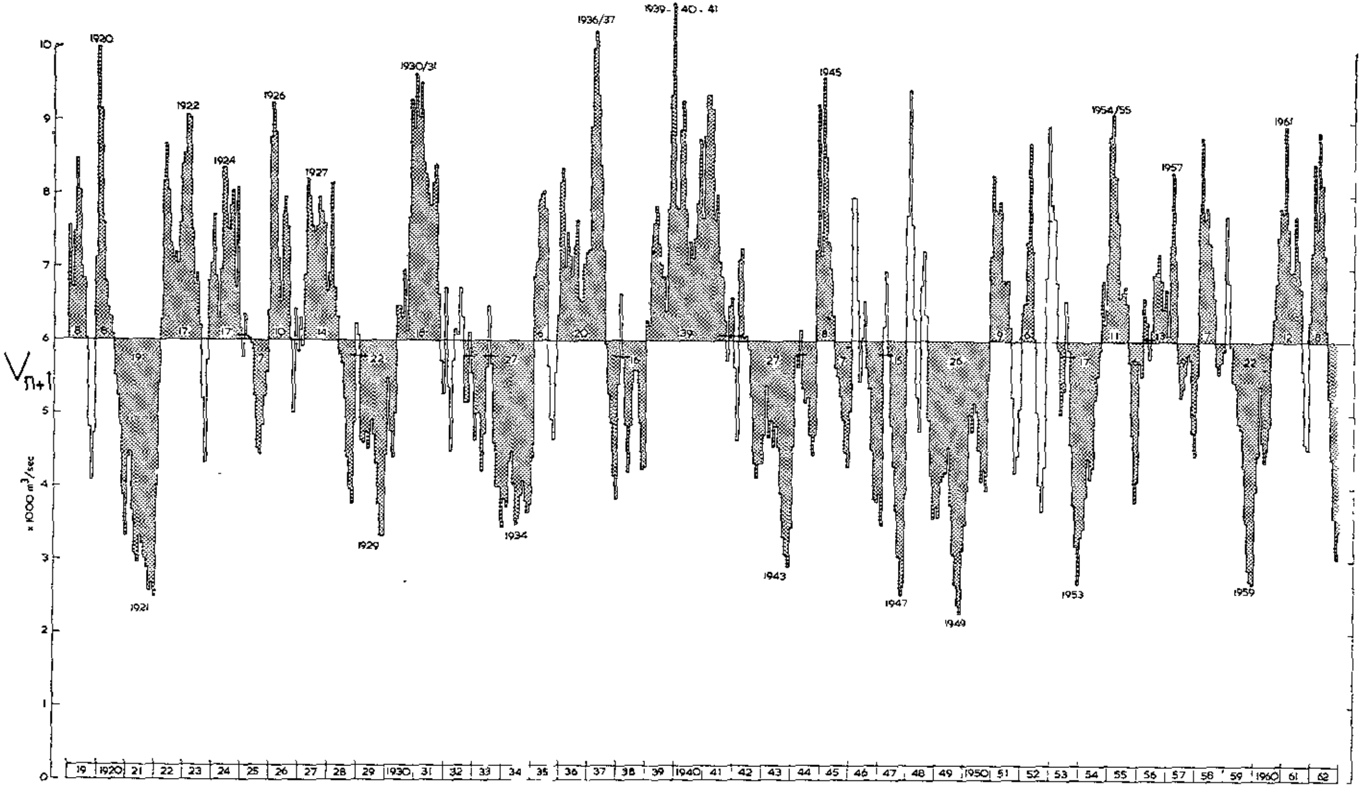

Fig. 4 — Time history of the total groundwater storage V_{n+1}, as calculated from the monthly mean runoff values of the Rhine, with $p = 1/3$ (mainly) and $r = 0{,}8$, for the space of time 1919-1962. All values in m³/sec.

It is important to state, that not in one single case there is opposition between the two performances. There is not any *H*-persistency in one figure coinciding with an *L*-persistency in the other figure or otherwise.

These facts lead to three important conclusions :

1. as to the essential character of persistency there is full correspondance between the presentation of fig. 1 and of fig. 4.

So the same terminology can be used for the periods of low groundwater table, as calculated by the PR-method.

2. the present investigation offers a much more accurate discernment, resulting in the discrimination of more persistencies and longer duration.

3. the PR-model touches the essence of the phenomenon of persistency, which will be discussed in the next paragraphs.

From this it looks justified to regard fig. 4 as a better source of information about persistencies than the previous fig. 1.

7. SOME FEATURES OF PERSISTENCIES

a) The persistencies prove to be irregular fluctuations in the basin storage curve. Everly periodicity is absent. A law of pure accident rules the succession of the individual ups and downs of the storage level, as well as the length of the individual *H*- or *L*-periods.

b) Of great importance is the long duration of many persistencies. Table 2 indicates a total number of 24 persistencies of a length of 12 months and more, up to 39 months as a maximum.

TABLE 2

24 persistencies of 12 and more months duration, in a 62-years period on the Rhine

year	length in months		year	(continued)	
	of *H*-persistencies	of *L*-persistencies		H	L
1	2	3	1	2	3
1902/03		15	1930/31	18	
1904/05		13	1933/34		27
1908/09		15	1936/37	20	
1910/11	17		1938		16
1912/13	12		1939/42	39	
1914/15	19		1942/44		27
1916/17	25		1946/47		15
1920/21		19	1948/50		26
1922/23	17		1953/54		17
1924/25	17		1956	13	
1927	14		1958/60		22
1928/30		22	1961	12	

It may be marked separately that there are 5 persistencies of more than two-year duration.

c) Without exception well-known periods of droughts are characterised by *L*-persistencies of long duration, wet years by *H*-persistencies of long duration.

d) If we maintain the definition to speak of persistencies if a *H*- or *L*-condition exists for at least 6 months at an uninterrupted stretch and to count the total duration over interruptions of 1 or 2 months if such interruptions are followed up or preceeded by at least 2 or 4 months, then we find for a period of 62 years for the Rhine :

TABLE 3

The occurrence of persistencies

	H persistencies	*L* persistencies
number of persistencies	24	20
total number of months enclosed	322	299
mean duration in months	13	15
maximum duration in months	39	27
% of time in condition of persistency	84%	

This table shows, that in 84% of the total time, that is in 10 of the 12 months in a year, the river Rhine is in a condition of persistency. So persistency is the dominating condition of the river.

Taking into account, that more than 50% of discharge consists of groundwater (see § 8) it may be clear, that any investigation concerning the relation between precipitation and runoff, must start with a preliminary research as introduced here.

e) The overall mean value of basin storage is found to be 6000 m^3/sec. The significance of this amount of water may be illustrated by mentioning that the lowest monthly mean runoff is 680 m^3/sec (1947). A netto precipitation over many months of only 20% of normal precipitation is less than ever stated. Under such an abnormal condition the total storage should provide for

$6000 : (680 - 2200/5) = 25$ months.

The groundwater storage would be fully exhausted by the mean discharge of the river in

$6000 : 2200 = 2{,}7$ months.

This total storage may rightly be called a predominating factor in the regime of a river.

f) The mean storage of 6000 m^3/sec is visibly not the "normal" or stable position of the groundwater table, as for a pendulum the lowest position of the weight. On the contrary. It is not normal for the groundwater table to be at any position whatever. Its normal habit is to move, to go up and to go down as caused by the excesses and deficiences of the meteorological fluctuations.

This essential feature is strongly illustrated by the curve of fig. 4.

g) As the basin storage capacity is high in relation to the meteorological fluctuations, it often takes many months to switch over from a state of excess into a state of deficiency vice versa. The true character of the storage is its integrating and smoothing function.

h) If there is a deficit in the storage, it is not possible to refill the storage up to its normal table by normal precipitation. On the contrary, it requires over normal precipitation during several months.

8. THE RELATIVE MAGNITUDE OF BASEFLOW

The PR-calculation model splits the actual runoff A up into its components "direct runoff" $(A - G)$ and base runoff G. As already has been emphasised there is no fixed relation between these two. On the contrary both values fluctuate within wide limits and are partly undependent from each other. So for good understanding of the dynamic character of baseflow in relation to direct flow in table 4 we brought together maxima and minima of each of the more important components.

To start with column 11 one sees that an almost constant part of precipitation P infiltrates into the ground and replenishes groundwater storage. In our model this ratio is found from (11) and (12) :

$$\frac{P_g}{P} = \frac{p \cdot r}{1 - r + p \cdot r} \tag{16}$$

For $r = 0{,}8$ and $p = 1/3$, being applicable for 85% of all months, this ratio is 0,57. For the highest monthly mean runoff, being 7350 m^3/sec in Jan. 1920, p acquires its lowest value 1/5,5 and accordingly the ratio P_g/P becomes 0,42.

On account of this almost constant ratio one should possibly expect a constant base runoff G. Of course this is not the case, owing to the variability of P, see column 9. Precipitation varies between 0 and 10320 m^3/sec, but G (column 3) varies between 460 and 2000 m^3/sec. So G is not at all constant and yet it does not fluctuate to the same degree as precipitation.

Direct runoff, column 4, fluctuates between 0 and 5930 m^3/sec. As one may observe the distribution of A over G and $(A - G)$ is highly fluctuating. This is demonstrated in column 10, giving the values of the ratio G/A. Its minimum value is found to be 12%, its maximum 100%. So it is quite clear, that there is no question of a constant ratio between river discharge and base runoff.

The mean ratio between base runoff and river discharge is equal to 55% : row 1 in table 4. The ratio P_g/P as a mean overall value has the same magnitude 55%. So our PR-calculation leads ot the outcome, that 55% of all precipitation takes the roundabout way over groundwater to come to runoff in the quality of base runoff.

But in an arbitrary chosen month base runoff varies in absolute sense as well in relative sense within the wide limits as presented in table 4.

9. INFLUENCE OF PRECEEDING MONTHS

The influence of preceeding months is essential for persistencies. So it deserves to be treated here somewhat more in detail.

After par. 2 the discharge in month $n + m$ is composed of a part, originating of month n, and all other parts. This may be formulated as follows :

$$A_{n+m} = r^m \left[G_n + p \cdot (A_n - G_n)\right] + \text{other components.}$$

Due to the reduction factor r^m the contribution of month n decreases as time strides along. It will be clear, that the relative importance of this distribution, being

$$\frac{r^m [G_n + p \cdot (A_n - G_n)]}{A_{n+m}} \tag{17}$$

will be varying endlessly and never can be expressed in general analytical terms. Of course it is possible to express the influence of month n on the runoff in month $n + m$ in terms of correlation and in regression coefficients. But this method does not reveal the true causal relations. The coefficients p and r however give us a tool to calculate continuously the influence of former runoffs on the present one and on future situations. This method may be regarded to be highly superior to correlation methods.

Fig. 5 demonstrates the procedure as well as the results of such a calculation.

TABLE 4

Some maxima and minima in a 62-year period of Rhine discharges. All values in m^3/sec

	A	G	$A-G$	p	G'	V_{n+1}	D	P	$G/A\%$	$P_g/P\%$
1	2	3	4	5	6	7	8	9	10	11
gen. MEAN	2200	1200	1000	1/3,3	1500	6000	0	2200	55%	55%
some maxima										
1920 1	**7350**	1420	**5930**	1/5,5	2500	10000	+2880	10230	19%	42%
1937 4	3980	**2000**	1980	1/3,5	**2560**	**10240**	+ 240	4220	50%	53%
1948 1	6520	780	5740	1/5,0	1930	7720	**+3800**	**10320**	*12%*	44%
Some minima										
1920 11	970	970	*0*	1/3,0	970	3880	− 970	*0*	**100%**	57%
1947 10	*680*	610	70	1/3,0	630	2520	− 520	160	90%	57%
1948 3	*1930*	*1890*	40	*1/3,0*	1900	7600	−*1840*	90	98%	57%
1949 11	750	480	270	1/3,0	*570*	*2280*	− 120	630	64%	57%
1949 12	1420	*460*	960	1/3,0	780	3120	+ 840	2260	32%	57%

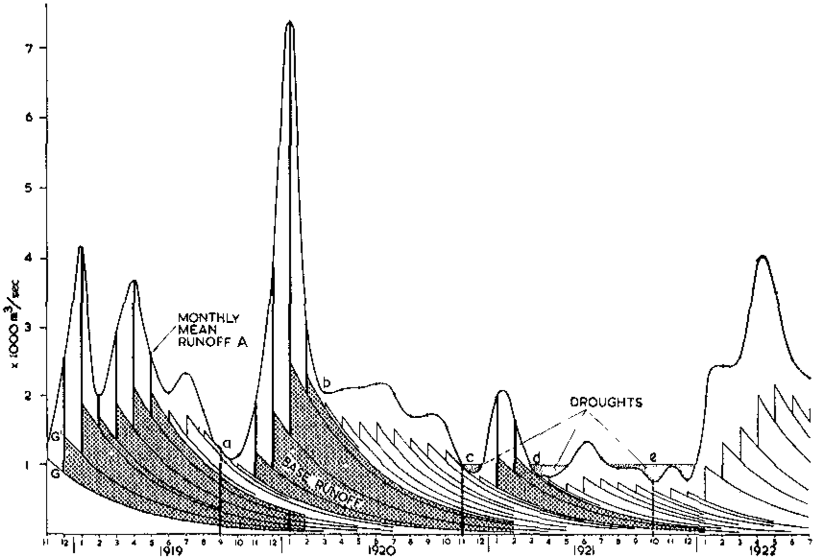

Fig. 5 — A runoff hydrograph of the Rhine 1919-1922 from monthly mean values. The corresponding values G and G' have been calculated with the PR-method. The falling e-curves are constructed with $T = 5$ months.

The influence of each individual month on the base runoff in following months is clearly visible. A short description may be of use.

a) The diagram contains three vernal seasons of highly different behaviour. So 1919 with high monthly runoffs in 1, 3, 4 and 5. This results in a high level of G and G'. The excess of groundwater storage of these months is part of a H-persistency of 8 months as can be seen in fig. 4.

b) The vernal season of 1920 shows only 3 months of high runoff. Though 1919 1 shows a high monthly mean runoff (the highest known value), this lasts too short and base runoff rapidly reduces.

c) In winter 1921 there are no high runoffs at all. Groundwater runoff, being already low in 11, 12 1920, is not increased in this winter. One may compare GG' in 4 1919, 4 1920 and 4 1921.

d) Fig. 4 shows an important L-persistency, beginning 8 1920 and lasting to 2 1922, so having a length of 19 months. Fig. 5 shows the details of this persistency, especially the significance of base runoff.

e) The large persistency 1921 shows 3 periods of well-known drought being marked *c*, *d* and *e*. *In 11 1920 and 3 1921 (c and d) the river runoff consists only of base*-flow.

f) In 9 1919 river runoff consists for 100% of base runoff as well. But thanks to the high level of base runoff the discharge was above the fatal mark.

g) This diagram shows clearly the criteria which have to be taken into account to determine p and r. As indicated already G must always be found below A or equal to A. In fig. 5 this sensitive criterium is active in *a*, *c* and *d* and nearly in *b*. By a way of trial and error fitting values of p and r have been found. It requires some 10 or 20 years as a minimum to meet periods being critical in this respect.

h) In fig. 5 the base runoff values, starting in the vernal seasons, have been shaded. One may observe the contribution of the high discharges of the winter months in the following autumn. In 9 1919 (marked a) this amounts to 60% of the runoff in that month, being only base runoff. In 11 1920 (point *c*) this contribution is 28% and in 10 1921 it is only 13%.

k) The PR-model allows to carry on the calculation up to the present moment. The latest found values G and G' are indeed the values as physically present at the latest moment. This is an advantage of the PR-model above the method of moving averages. The latter one has its centre some months ago and the calculated mean value is a notion, it has no real physical existance.

l) In many publications base runoff is presented as being more or less constant and even the term "base runoff" suggests this. But fig. 5 demonstrates, that base runoff is the outcome of a most dynamic process. "Old" groundwater is constantly running to the river and the groundwater table is from month to month replenished with "new" groundwater. From this results the saw-toothed appearance of the curve GG'.

If one replaces these saw-tooth by a smoothed curve or envelope the real dynamic character of base runoff is hidden. Deliberately this procedure has not been used here in order not to lose an important feature.

m) Sometimes baseflow is supposed to be something like the envelope of the runoff troughs. It proves already to be difficult to draw an evelope of the really low troughs, but for the interjacent months it is practically impossible to do so. The PR-model provides a possibility to determine the desired curve with more objective means.

n) The diagram demonstrates the importance of knowledge about base runoff, especially in relation to the occurrence of droughts. It makes clear, that droughts only come to existance by interaction of small precipitation and low base runoff. Fig. 5 shows clearly, that it is impossible to obtain droughts in a short space of time after high discharges. From a statistical analysis (being too extensive to be reproduced in this paper) it has become evident, that after some three or four months of G'-values below the general mean the chance of a period of drought increases rapidly. Knowledge of the actual magnitude of base runoff may be regarded as imperative for all forecasting work. Perhaps the PR-model as developed may contribute to that knowledge.

10. THE STORAGE FUNCTION OF THE RIVER BASIN

Fig. 4 gives the height of basin storage from month to month. The overall mean value over 62 years is about 6000 m^3/sec. Without replenishment this amount should give a decreasing base runoff, being in successive months :

1200, 960, 768, 614, 491, 393, 314, 252, 201, 161,...... m^3/sec.

The sum is 6000. The total amount of water represented by this number is :

$6000\ m^3/sec \times 2{,}63 .\ 10^6\ sec/mth \times 1\ mth = 16 .\ 10^9\ m^3$.

The catchment of the Rhine amounts to 165.000 km^2. So the total storage is equivalent to a netto watersheet over the whole catchment of

$16 .\ 10^9\ m^3 : 165.000 .\ 10^6\ m^2 = 0{,}1\ m$.

If we take a storativity factor of 0,1 à 0,2, the groundwater sheet has as a matter of fact a thickness of 1/2 à 1 m.

This result looks quite acceptable. It fully agrees with a communication of DREIBELBIS in the Journal of geophysical research Aug. 1962 p. 3425.

The thickness of the groundwatersheet will vary in space as well as in time. Variation in space will be caused by diversity in slope, in vegetation and especially in the composition of the ground. The mean value of thickness of the groundwatersheet is so small, that there is space enough for the required variability.

Variation in time is presented by fig. 4. If we hold the vertical scale for representing the groundwater table and every 1000 m^3/sec as representing 1 dm, the height is found to vary between about 2,5 dm and 10,5 dm. Of course this is a first grasp only to present an order of magnitude and to make us familiar with the implications of base runoff in a quantitative sense.

The Swiss lakes indicate a variation in the order of magnitude of 1 m. Having a total area of 1219 km^2, these lakes represent only a storage capacity of 8% of the total capacity as calculated after the PR-method. If we include the proper area of the river itself and its tributaries it only represents 10% of the required area.

So it may be clear that persistencies are caused practically exclusively by the storage capacity of the catchment, lakes and the proper riverbed being only minor parts of this total capacity.

In this respect may be mentioned too the storage capacity of the annual snow-covering. Properly the subject snow should require a third paper. Suffice here to say, that snow plays no role at all for the catchment downstream Basel, so for 80% of the whole basin. So even if snow should play an important role in the upper region of the Swiss mountains, it can only contribute some non-essential percents to the indicated total storage.

Moreover, if snow melts away in spring it increases the actual runoff *A*. The method used here does not discriminate between new precipitation and the melting of old precipitation. So the effect of snow melting is equal to a delay of precipitation, being principally different from storage and running off in later months. This consideration leads to the conclusion that snow melting does not contribute anything to the essence of persistencies. Of course it may contribute to the occurrence of persistencies, for instance a small amount of snow covering at the end of a winter tends to smaller discharges in spring and to promote *L*-persistency, in the same way as *L*-persistency is promoted by small precipitation.

This influence of snow will however be very small as already has been stated in the Helsinki paper. And the glaciers, being only 1,8 % of the Swiss catchment, have no measurable influence.

We described the storage of the catchment as a sheet of groundwater with an ever varying thickness. As an average it may vary between 2,5 dm and 10,5 dm thickness (0,5 dm demonstrates here not accuracy but refers to the description, given above). From fig. 4 we may conclude, that never in the 62-year period groundwater storage has been exhausted entirely, its lowest value being 40% of the general mean height or 20% of the maximum height.

If we speak however of a varying sheet of storage, then we must accept a zero level of this sheet, though it never occurs, that groundwater storage falls back to this zero level. We may suppose, that below this zero level there is at many areas a large amount of "deeper" groundwater. In our supposition these deeper sheets of groundwater may participate in the general movement from storage to the river. But the total volume will never change and this constant part of groundwater does not contribute to essence, nor to occurrence of persistencies.

11. CONCLUSION

Persistency is the condition of a river of a high or low discharge for an unusual sequence of months at a stretch, being at least six months.

The river Rhine proves to be in 84% of the time in a condition of persistency; lengths of more than two years occur frequently. Periods of droughts are identical with *L*-persistencies of long duration. These occurences can be fully explained by groundwater storage. About 57% of precipitation is feeding base storage, which comes to runoff gradually in following months. The groundwatersheet may have a thickness of 1/2 à 1 m as a mean over the whole catchment. The actual height of groundwater table may vary between 50% and 150% of the general mean height.

The general character of the interaction between netto precipitation, storage and runoff could be explained by a special developed model, called PR-model, which may be of use for forecasting and other specific investigations.