

Hy-167

*Ir. J. W. van der Made*  
Rijkswaterstaat, Netherlands

**A dimensionless representation of flood wave subsidence  
by means of a subsidence function**

**EXTRACT OF PUBLICATION N° 81**

**SYMPOSIUM OF TUCSON 1968**

**INTERNATIONAL ASSOCIATION OF SCIENTIFIC HYDROLOGY**

# A dimensionless representation of flood wave subsidence by means of a subsidence function

Ir. J.W. van der Made  
Rijkswaterstaat, Netherlands

**SUMMARY:** By applying Forchheimer's formula for flood wave crest subsidence in a discharge profile with partly or entirely stream carrying flood plains, a relation has been developed between the subsidence of a unit runoff in a unit profile and the subsidence of a variable runoff in the profile considered. Numerical values of a "subsidence function" were derived from a computer calculation. Observations of an actual situation fit reasonably the theoretical scheme.

**RÉSUMÉ:** En appliquant la formule de Forchheimer à l'atténuation des crêtes des crues, qui passent un profil, composé d'un lit mineur et d'un lit majeur, une relation a été trouvée entre l'atténuation d'un débit unitaire dans un profil unitaire et l'atténuation d'un débit variable dans le profil examiné. Des valeurs numériques d'une «fonction de l'atténuation» ont été déduites d'un calcul avec ordinateur électrique. En appliquant le modèle théorique à une situation réelle établie par des observations, l'on trouve une concordance satisfaisante.

## 1. INTRODUCTION

For the crest attenuation of a flood wave on a river reach of a limited length Forchheimer's formula produces a satisfactory result [1]. This formula reads as follows:

$$\frac{dQ_t}{dx} = \frac{B_b^2 Q_t}{2S_b (dQ_t/dy)^3} \cdot \frac{\partial^2 Q_t}{\partial t^2} \tag{1}$$

where:

- $dQ_t/dx$  the change of the peak runoff along the river reach;
- $B_b$  the storing width of the waterlevel;
- $S_b$  the bottom slope;
- $Q_t$  the peak runoff;
- $dQ_t/dy$  the runoff change per depth unit (= the slope of the tangent to the stage-discharge relationship);
- $\partial^2 Q_t/\partial t^2$  the curvature at the peak of the runoff hydrograph.

For given cross-sectional area and river slope the variables are the peak runoff  $Q_t$  and the peak curvature  $\partial^2 Q_t/\partial t^2$  respectively.

Supposing a certain relation between these variables, for instance a proportionality, the crest attenuation can be regarded as depending on the peak runoff  $Q_t$  only.

In a previous publication [2] the following proportionality was applied:

$$\frac{\partial^2 Q_t}{\partial t^2} = R \cdot Q_t \tag{2}$$

which led to the formula:

$$\frac{dQ_t}{dx} = \frac{B_b^2 Q_t^2}{2S_b (dQ_t/dy)^3} \cdot R \tag{3}$$

**2. APPLICATION TO THE RECTANGULAR DISCHARGE PROFILE**

In a rectangular profile, the runoff amounts, in accordance with Manning, to:

$$Q_t = B_s K \sqrt{S_b} \cdot y^{5/3} \tag{4}$$

where:

- $B_s$  the stream carrying width;
- $K$  Manning's coefficient;
- $y$  the waterdepth.

Applying formula (3) we find a crest subsidence:

$$\frac{dQ_t}{dx} = - \frac{27}{250} \cdot \frac{B_b^2 \cdot R}{S_b (B_s K \sqrt{S_b})^{9/5}} \cdot Q_t^{4/5} \tag{5}$$

Now introduce a unit profile (see fig. 1), in which the storing width equals the stream carrying width ( $B_b = B_s$ ) and which at a unit depth  $h$  is carrying a unit runoff  $Q_f$ .

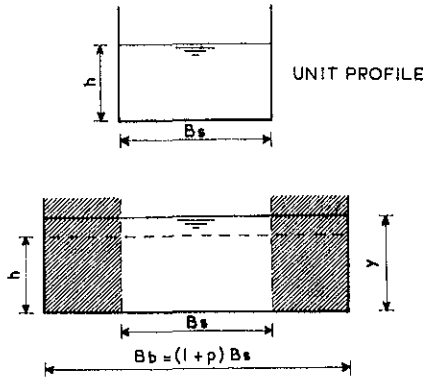


FIGURE 1

Here:

$$Q_f = B_s K \sqrt{S_b} \cdot h^{5/3} \tag{6}$$

and:

$$\left(\frac{dQ_t}{dx}\right)_f = - \frac{27}{250} \cdot \frac{B_s^2 \cdot R}{S_b (B_s K \sqrt{S_b})^{9/5}} \cdot Q_f^{4/5} \tag{7}$$

We introduce:

$$dq = \left(\frac{dQ_t}{dx}\right) / \left(\frac{dQ_t}{dx}\right)_f \tag{8}$$

$$q = Q_t / Q_f \tag{9}$$

$$B_b = (1 + p) B_s \tag{10}$$

The relative attenuation  $dq$  of the crest in an arbitrary rectangular profile is the ratio between (5) and (7):

$$dq = (1 + p)^2 \cdot q^{4/5} \tag{11}$$

where a constant  $R$  value (see formula (2)) is assumed.

The relation between  $dq$  and  $q$  can be represented graphically on double log paper by a straight line with a slope  $4/5$ . See figure 2.

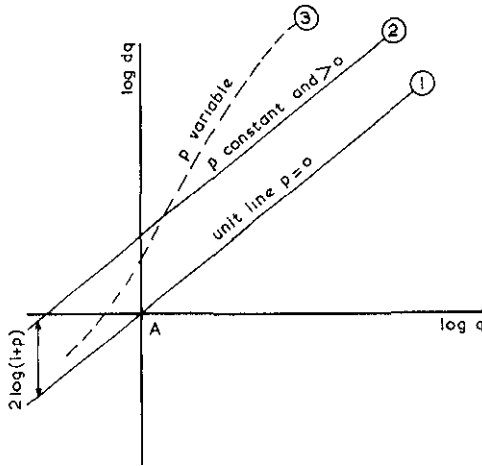


FIGURE 2

The unit profile line 1 ( $B_b = B_s$ ;  $p=0$ ) leads through the origin  $A$ . For a profile with a constant storing width, exceeding the stream carrying width ( $B_b > B_s$ ;  $p > 1$ ) the line (line 2) is situated at a distance  $2 \log(1 + p)$  above the unit line 1. A varying storing width produces an accordingly varying distance, which leads to a curved line (line 3).

Attention must be paid to the fact that only the storing width at the water level is of importance. So the foregoing is also applicable to flood plains where no longitudinal flow is assumed to occur (fig. 3).

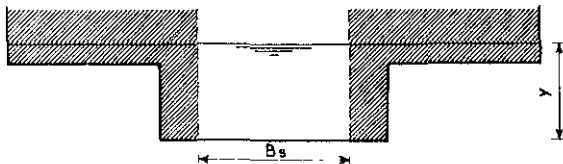


FIGURE 3

We actually have to do here with a rectangular discharge profile with a variable storing width.

### 3. APPLICATION TO A PROFILE WITH LONGITUDINAL FLOW OVER THE FLOOD PLAINS

In a profile with partly or entirely longitudinal flow over the flood plains the runoff amounts to:

$$Q_t = Q(\text{low water bed}) + Q(\text{flood plains}) \tag{12}$$

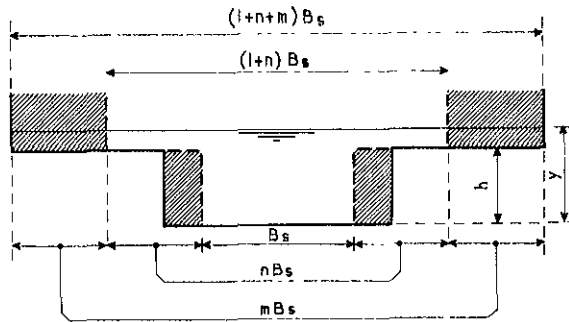


FIGURE 4

Now suppose the stream carrying part of the low water bed to be the unit profile mentioned before and the runoff in this profile, when the water level equals the flood plain level, to be the unit runoff  $Q_f$ .

From figure 4 the ratio between the storing width  $B_b$  and the unit width  $B_s$  is derived:

$$\begin{aligned} B_b &= B_s + nB_s + mB_s \\ &= B_s(1+n+m) \end{aligned} \tag{13}$$

Here  $nB_s$  is the width of the part of the flood plains with longitudinal flow, whereas  $mB_s$  is the width of the part without such flow.

Now:

$$Q_t = B_s K \sqrt{S_b} \cdot y^{5/3} + nB_s K \sqrt{S_b} \cdot (y-h)^{5/3} \tag{14}$$

from which follows:

$$Q_t = Q_f \cdot \left\{ \left( \frac{y}{h} \right)^{5/3} + n \left( \frac{y}{h} - 1 \right)^{5/3} \right\} \tag{15}$$

and:

$$q = \left( \frac{y}{h} \right)^{5/3} + n \left( \frac{y}{h} - 1 \right)^{5/3} \tag{16}$$

The crest subsidence follows from substitution of (14) into (3):

$$\frac{dQ_t}{dx} = -\frac{27}{250} \cdot \frac{B_b^2 R}{S_b(B_s K \sqrt{S_b})} \cdot h^{4/3} \cdot \frac{\left\{ \left(\frac{y}{h}\right)^{5/3} + n \left(\frac{y}{h} - 1\right)^{5/3} \right\}^2}{\left\{ \left(\frac{y}{h}\right)^{2/3} + n \left(\frac{y}{h} - 1\right)^{2/3} \right\}^3} \quad (17)$$

From (6) follows:

$$h^{4/3} = \left( \frac{Q_f}{B_s K \sqrt{S_b}} \right)^{4/5} \quad (18)$$

Substitution into (16) produces:

$$\frac{dQ_t}{dx} = -\frac{27}{250} \cdot \frac{B_b^2 R}{S_b(B_s K \sqrt{S_b})^{9/5}} \cdot Q_f^{4/5} \cdot \frac{\left\{ \left(\frac{y}{h}\right)^{5/3} + n \left(\frac{y}{h} - 1\right)^{5/3} \right\}^2}{\left\{ \left(\frac{y}{h}\right)^{2/3} + n \left(\frac{y}{h} - 1\right)^{2/3} \right\}^3} \quad (19)$$

This formula partly corresponds with the subsidence formula (7) of the unit profile. Now:

$$\frac{dQ_t}{dx} = \left( \frac{dQ_t}{dx} \right)_f \cdot \left( \frac{B_b}{B_s} \right)^2 \cdot \frac{\left\{ \left(\frac{y}{h}\right)^{5/3} + n \left(\frac{y}{h} - 1\right)^{5/3} \right\}^2}{\left\{ \left(\frac{y}{h}\right)^{2/3} + n \left(\frac{y}{h} - 1\right)^{2/3} \right\}^3} \quad (20)$$

Here we introduce the "subsidence function":

$$sf = \frac{\left\{ \left(\frac{y}{h}\right)^{5/3} + n \left(\frac{y}{h} - 1\right)^{5/3} \right\}^2}{\left\{ \left(\frac{y}{h}\right)^{2/3} + n \left(\frac{y}{h} - 1\right)^{2/3} \right\}^3} \quad (21)$$

Then, from (20), (8), (13) and (21) follows:

$$dq = (1+n+m)^2 \cdot sf \quad (22)$$

#### 4. DISCUSSION ON THE SUBSIDENCE FUNCTION

Before investigating the trend of the subsidence values in detail, the subsidence function will be discussed. The numerical values of this function were calculated by means of a computer for 20 values of the ratio  $y/h$  (going from 1.05 up to and including 2.00 with intervals 0.05) in 25 profiles with different  $n$  values (going from 0 up to and including 12.0 with intervals 0.5). The relative runoff  $q$  (formula (16)) was calculated for the same cases. So one thousand calculations were executed to investigate the shape and the trend of the subsidence function. This could only reasonably be achieved using a computer. The results are stated in table 1. They have been graphically represented in figure 5. The general trend can be seen in figure 6, line 1.

TABLE

$\frac{y}{h}$	1.05		1.10		1.15		1.20		1.25		1.30		1.35		1.40		1.45		1.50	
	q	sf	q	sf	q	sf	q	sf	q	sf	q	sf	q	sf	q	sf	q	sf	q	sf
0	1.08	1.07	1.17	1.14	1.26	1.20	1.36	1.28	1.45	1.35	1.55	1.42	1.65	1.49	1.75	1.57	1.86	1.64	1.97	1.72
0.5	1.09	.887	1.18	.866	1.28	.866	1.39	.878	1.50	.897	1.62	.921	1.74	.949	1.86	.980	1.99	1.01	2.12	1.05
1.0	1.09	.746	1.19	.678	1.30	.648	1.42	.636	1.55	.636	1.68	.643	1.82	.655	1.97	.671	2.12	.690	2.28	.712
1.5	1.09	.634	1.20	.542	1.33	.499	1.46	.480	1.60	.473	1.75	.473	1.91	.480	2.08	.490	2.25	.503	2.44	.518
2.0	1.10	.543	1.22	.441	1.35	.395	1.49	.373	1.65	.364	1.82	.363	2.00	.367	2.19	.374	2.39	.385	2.60	.397
2.5	1.10	.470	1.23	.364	1.37	.319	1.53	.298	1.70	.289	1.88	.288	2.08	.291	2.29	.297	2.52	.305	2.75	.316
3.0	1.11	.409	1.24	.305	1.39	.263	1.56	.243	1.75	.235	1.95	.234	2.17	.236	2.40	.242	2.65	.249	2.91	.258
3.5	1.11	.358	1.25	.258	1.41	.219	1.59	.202	1.80	.195	2.02	.194	2.26	.197	2.51	.202	2.78	.208	3.07	.217
4.0	1.11	.316	1.26	.221	1.43	.186	1.63	.170	1.85	.164	2.09	.164	2.34	.166	2.62	.171	2.91	.178	3.23	.185
4.5	1.12	.280	1.27	.191	1.45	.159	1.66	.146	1.90	.141	2.15	.140	2.43	.143	2.73	.148	3.05	.154	3.38	.161
5.0	1.12	.250	1.28	.166	1.47	.138	1.70	.126	1.95	.122	2.22	.122	2.52	.125	2.84	.129	3.18	.135	3.54	.141
5.5	1.12	.223	1.29	.146	1.50	.120	1.73	.110	2.00	.107	2.29	.107	2.61	.110	2.95	.114	3.31	.119	3.70	.126
6.0	1.13	.201	1.30	.129	1.52	.106	1.77	.097	2.05	.094	2.36	.095	2.69	.098	3.05	.102	3.44	.107	3.86	.113
6.5	1.13	.181	1.31	.115	1.54	.094	1.80	.086	2.10	.084	2.42	.085	2.78	.088	3.16	.092	3.58	.096	4.01	.102
7.0	1.13	.164	1.32	.103	1.56	.084	1.83	.077	2.14	.075	2.49	.076	2.87	.079	3.27	.083	3.71	.088	4.17	.093
7.5	1.14	.149	1.33	.092	1.58	.075	1.87	.069	2.19	.068	2.56	.069	2.95	.072	3.38	.076	3.84	.080	4.33	.085
8.0	1.14	.136	1.34	.083	1.60	.068	1.90	.063	2.24	.062	2.62	.063	3.04	.066	3.49	.070	3.97	.074	4.49	.079
8.5	1.14	.125	1.36	.076	1.62	.062	1.94	.057	2.29	.056	2.69	.058	3.13	.061	3.60	.064	4.10	.068	4.64	.073
9.0	1.15	.115	1.37	.069	1.64	.056	1.97	.052	2.34	.052	2.76	.053	3.21	.056	3.71	.059	4.24	.063	4.80	.068
9.5	1.15	.105	1.38	.063	1.66	.051	2.00	.048	2.39	.048	2.83	.049	3.30	.052	3.82	.055	4.57	.059	4.96	.063
10.0	1.15	.097	1.39	.058	1.69	.047	2.04	.044	2.44	.044	2.89	.046	3.39	.048	3.92	.052	4.50	.055	5.12	.059
10.5	1.16	.090	1.40	.053	1.71	.043	2.07	.041	2.49	.041	2.96	.043	3.47	.045	4.03	.048	4.63	.052	5.27	.056
11.0	1.16	.083	1.41	.049	1.73	.040	2.11	.038	2.54	.038	3.03	.040	3.56	.042	4.14	.045	4.76	.049	5.43	.053
11.5	1.16	.077	1.42	.045	1.75	.037	2.14	.035	2.59	.036	3.09	.037	3.65	.040	4.25	.043	4.90	.046	5.59	.050
12.0	1.17	.072	1.43	.042	1.77	.035	2.18	.033	2.64	.034	3.16	.035	3.73	.038	4.36	.041	5.03	.044	5.75	.047

1

1.55		1.60		1.65		1.70		1.75		1.80		1.85		1.90		1.95		2.00	
q	sf	q	sf	q	sf	q	sf	q	sf	q	sf	q	sf	q	sf	q	sf	q	sf
2.08	1.79	2.19	1.87	2.30	1.95	2.42	2.03	2.54	2.11	2.66	2.19	2.79	2.27	2.91	2.35	3.04	2.44	3.17	2.52
2.26	1.09	2.40	1.13	2.55	1.17	2.70	1.21	2.85	1.25	3.01	1.30	3.17	1.34	3.33	1.39	3.50	1.44	3.67	1.48
2.45	.736	2.62	.761	2.79	.788	2.97	.816	3.16	.845	3.35	.876	3.55	.907	3.75	.939	3.96	.972	4.17	1.01
2.63	.535	2.83	.554	3.04	.575	3.25	.596	3.47	.618	3.70	.642	3.93	.666	4.17	.691	4.42	.716	4.67	.743
2.81	.411	3.04	.426	3.28	.442	3.53	.460	3.78	.478	4.04	.497	4.31	.517	4.59	.537	4.88	.558	5.17	.580
3.00	.327	3.26	.340	3.52	.354	3.80	.369	4.09	.385	4.39	.401	4.69	.418	5.01	.435	5.34	.453	5.67	.472
3.18	.269	3.47	.280	3.77	.292	4.08	.305	4.40	.319	4.73	.333	5.08	.348	5.43	.363	5.80	.379	6.17	.395
3.37	.226	3.68	.236	4.01	.247	4.35	.259	4.71	.271	5.08	.284	5.46	.297	5.85	.310	6.26	.324	6.67	.338
3.55	.194	3.90	.203	4.25	.213	4.63	.223	5.02	.234	5.42	.246	5.84	.258	6.27	.270	6.72	.282	7.17	.295
3.74	.169	4.11	.177	4.50	.186	4.90	.196	5.33	.206	5.77	.216	6.22	.227	6.69	.238	7.17	.249	7.67	.261
3.92	.149	4.32	.156	4.74	.165	5.18	.174	5.64	.183	6.11	.193	6.60	.202	7.11	.213	7.63	.223	8.17	.234
4.11	.132	4.54	.140	4.99	.148	5.46	.156	5.95	.164	6.46	.173	6.98	.182	7.53	.192	8.09	.201	8.67	.211
4.29	.119	4.75	.126	5.23	.133	5.73	.141	6.26	.149	6.80	.157	7.36	.166	7.95	.175	8.55	.184	9.17	.193
4.48	.108	4.96	.115	5.47	.121	6.01	.129	6.57	.136	7.14	.144	7.75	.152	8.37	.160	9.01	.168	9.67	.177
4.66	.099	5.18	.105	5.72	.111	6.28	.118	6.88	.125	7.49	.132	8.13	.140	8.79	.148	9.47	.155	10.2	.163
4.85	.091	5.39	.096	5.96	.103	6.56	.109	7.18	.116	7.83	.122	8.51	.130	9.21	.137	9.93	.144	10.7	.152
5.03	.084	5.60	.089	6.21	.095	6.84	.101	7.49	.107	8.18	.114	8.89	.121	9.63	.127	10.4	.135	11.2	.142
5.21	.078	5.82	.083	6.45	.089	7.11	.094	7.80	.100	8.52	.106	9.27	.113	10.0	.119	10.8	.126	11.7	.133
5.40	.072	6.03	.078	6.69	.083	7.39	.088	8.11	.094	8.87	.100	9.65	.106	10.5	.112	11.3	.118	12.2	.125
5.58	.068	6.24	.073	6.94	.078	7.66	.083	8.42	.088	9.21	.094	10.0	.100	10.9	.106	11.8	.112	12.7	.118
5.77	.064	6.46	.068	7.18	.073	7.94	.078	8.73	.083	9.56	.089	10.4	.094	11.3	.100	12.2	.106	13.2	.112
5.95	.060	6.67	.064	7.43	.069	8.22	.074	9.04	.079	9.90	.084	10.8	.089	11.7	.095	12.7	.100	13.7	.106
6.14	.057	6.88	.061	7.67	.065	8.49	.070	9.35	.075	10.2	.080	11.2	.085	12.1	.090	13.1	.095	14.2	.101
6.32	.054	7.10	.058	7.91	.062	8.77	.067	9.66	.071	10.6	.076	11.6	.081	12.6	.086	13.6	.091	14.7	.096
6.51	.051	7.31	.055	8.16	.059	9.04	.063	9.97	.068	10.9	.072	11.9	.077	13.0	.082	14.1	.087	15.2	.092



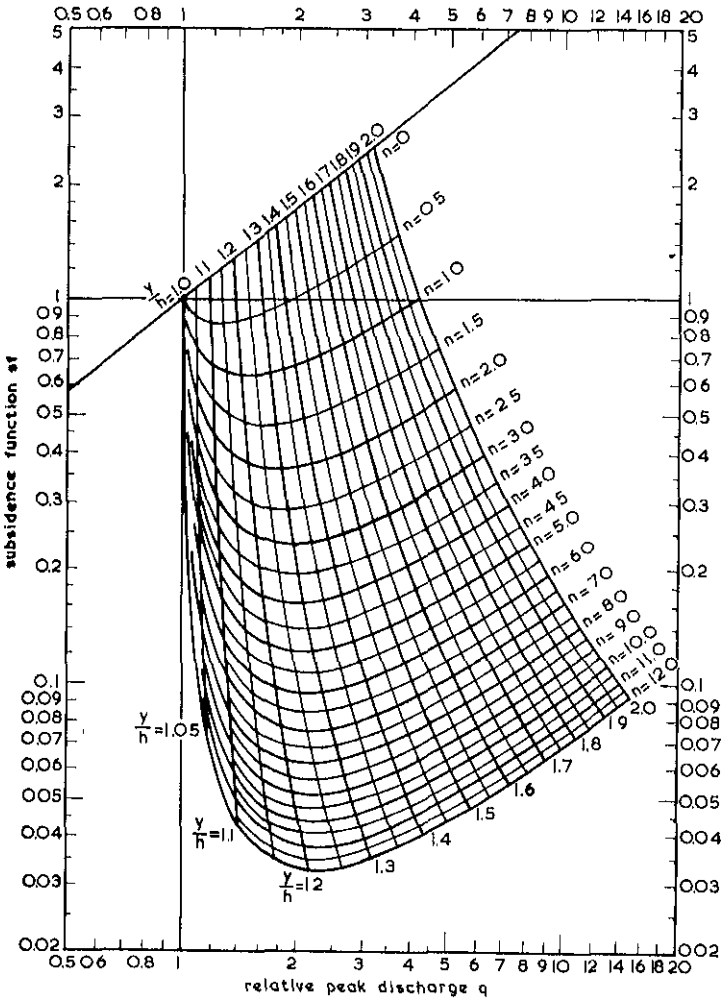


FIGURE 5

In the first place two special cases will be discussed:

- 1) The flood plains do not carry any flow over their entire width ( $n=0$ ).  
From (21) follows:

$$sf = \left(\frac{y}{h}\right)^{4/3} \tag{23}$$

and from (16):

$$q = \left(\frac{y}{h}\right)^{5/3} \tag{24}$$

These two formulæ lead to:

$$sf = q^{4/5} \tag{25}$$

This relation is represented on double log paper by a straight line with a slope 4/5 (line 2). It appears that the conception "subsidence function" is valid for the profiles with a constant streamcarrying width too (fig. 1 and 3).

2) The water depth goes to infinite:

From (21):

$$\lim_{y \rightarrow \infty} sf = \frac{1}{1+n} \left(\frac{y}{h}\right)^{4/3} \tag{26}$$

and from (16):

$$\lim_{y \rightarrow \infty} q = (1+n) \left(\frac{y}{h}\right)^{5/3} \tag{27}$$

This produces an expression in  $q$  for line  $sf$ :

$$\lim_{q \rightarrow \infty} sf = \frac{1}{(1+n)^{9/5}} \cdot q^{4/5} \tag{28}$$

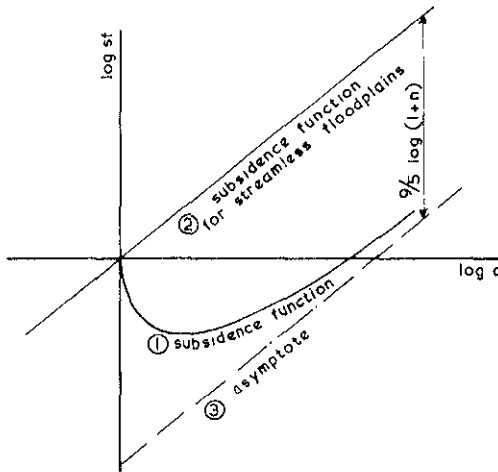


FIGURE 6

This represents the asymptote to which the subsidence function approaches when the runoff is increasing. In figure 6 it appears as the straight line 3, parallel to the line 2 and at a distance  $9/5 \log(1+n)$  below it.

**5. THE GENERAL TREND OF THE RELATION SUBSIDENCE-PEAK RUNOFF**

Now the relative subsidence  $dq$  follows from multiplication of  $sf$  by the square of the relative storing width  $(1+n+m)$ . Compare formula (22). In the graphical representation the  $sf$ -curve is shifted vertically over a distance  $2 \log (1+n+m)$ .

This produces figure 7.

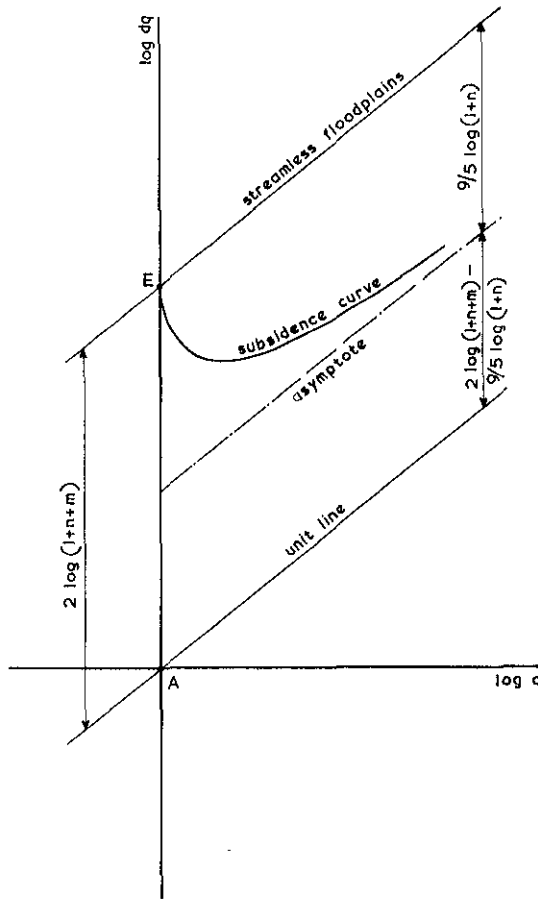


FIGURE 7

The vertical distance between the unit line and the asymptote amounts to  $2 \log (1+n+m) - 9/5 \log (1+n)$ . If the flood plains carry discharge over their entire width ( $m=0$ ) this distance becomes  $1/5 \log (1+n)$ .

Consequently it is possible to establish a general trend of the crest attenuation as a function of the peak runoff. In figure 8 a profile with flood plains is represented, together with subsidence graphs.

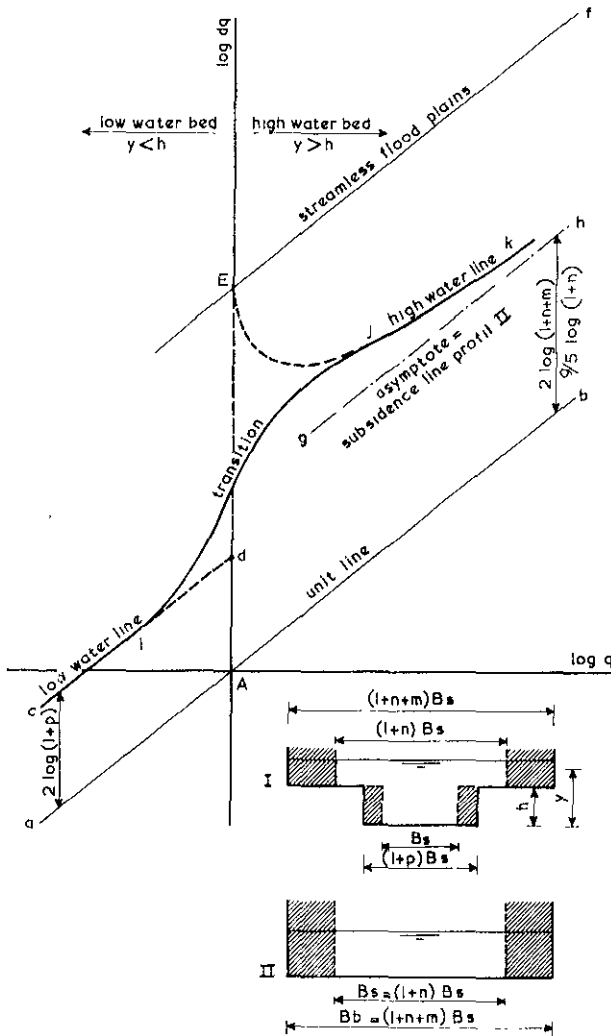


FIGURE 8

It can be noticed:

- 1) In the low water bed the stream carrying width amounts to  $B_s$ , the non stream carrying, storing width to  $(1+p)B_s$ . The values of the relative subsidence fit the straight line  $c-l-d$ , situated at a distance  $2 \log(1+p)$  above the unit line  $a-A-b$ .
- 2) At the flood plain level the subsidence jumps to the point  $E$ , at a distance  $2 \log(1+n+m)$  above the origin  $A$ . If there would be no discharge over the plains the relation would continue according to the line  $E-f$ , parallel to the unit line.
- 3) In discharge carrying flood plains however, the subsidences fall to lower values immediately above the plain level. This is due to the increasing of the flow profile above the flood plains through which an increasing part of the total water yield is transported. This hamper's storage of water for any considerable length of time.

- 4) The increasing sharpness of the higher flood waves causes a gradual turning off of the curve. This leads to a minimum subsidence value at a certain discharge.
- 5) For higher discharges a new increase of the subsidence follows. Gradually the conditions become more similar to those in a rectangular profile.
- 6) For very large floods the subsidence value in the profile with plains approaches the value in a rectangular profile like profile II in figure 8.

Here the stream carrying width amounts to:

$$B_s^* = (1 + n) B_s \tag{29}$$

whereas the storing width can be expressed by:

$$B_b^* = (1 + n + m) B_s \tag{30}$$

Apparently for very large floods the configuration of the profile at lower levels has no longer any bearing on the crest subsidence. This appears as follows:

The unit-runoff in the new rectangular profile amounts to:

$$Q_f^* = (1 + n) Q_f \tag{31}$$

Now we can derive the crest subsidence according (7):

$$\begin{aligned} \left(\frac{dQ_t}{dx}\right)_f^* &= -\frac{27}{250} \frac{B_s^{*2} \cdot R}{S_b (B_s^* K \sqrt{S_b})^{9/5}} \cdot Q_f^{*4/5} \\ &= \frac{27}{250} \frac{B_s^2 (1+n)^2 R}{S_b (B_s K \sqrt{S_b})^{9/5} \cdot (1+n)^{9/5}} \cdot (1+n)^{4/5} Q_f^{4/5} \\ &= (1+n) \left(\frac{dQ_t}{dx}\right)_f \end{aligned} \tag{32}$$

For discharges  $Q_t$ , different from the new unit runoff, holds (compare formula (11)):

$$\left(\frac{dQ_t}{dx}\right) / \left(\frac{dQ_t}{dx}\right)_f^* = \left(\frac{B_b^*}{B_s^*}\right)^2 \cdot \left(\frac{Q_t}{Q_f^*}\right)^{4/5} \tag{33}$$

From substitution of (32), (29), (30) and (31) follows:

$$\left(\frac{dQ_t}{dx}\right) / (1+n) \left(\frac{dQ_t}{dx}\right)_f = \left(\frac{1+n+m}{1+n}\right)^2 \cdot \left(\frac{t}{(1+n) Q_f}\right)^{4/5} \tag{34}$$

which leads to:

$$dq = \frac{(1+n+m)^2}{(1+n)^{9/5}} \cdot q \tag{35}$$

This expression also represents the asymptote to which the subsidence is approaching for very large floods. Under these conditions there is no difference between the two profile types.

## 6. COMPARISON BETWEEN THE THEORETICAL MODEL AND AN ACTUAL SITUATION

In actual situations the results will deviate to some extent from the theoretical scheme derived before. Especially the peak at *E* in figure 8 cannot be expected to be found in practice. This is a consequence of:

- 1) Irregularities in the cross section. Generally the flood plains do not reach their full width at one level. Besides they will not always have the same level at the two banks (fig. 9).

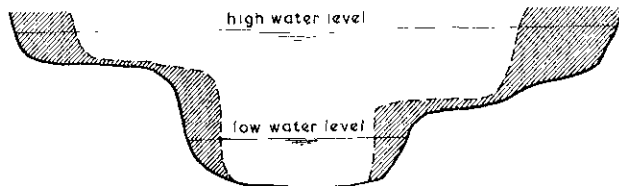


FIGURE 9

- 2) Irregularities in the longitudinal section. With regard to the river bed the flood plain level is not constant along the river (fig. 10).

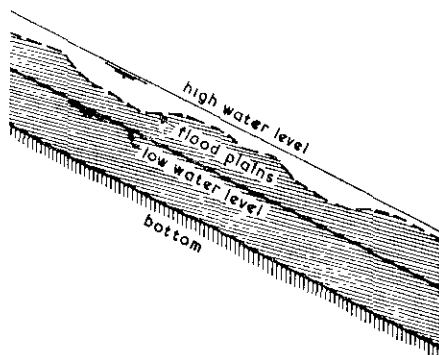


FIGURE 10

- 3) The fact that the foregoing theory is related to one point and not to a river reach of a certain length.

This is of particular importance when the subsidence trend along the section shows discontinuities, which is the case for instance when passing the flood plain level. The crest subsidence values, derived from the differences between the peak discharges at the upstream and the downstream end of the section considered respectively, always give something like a mean value. This ultimately results in a flattening of the peak at *E*.

- 4) The disturbing factor of tributary discharges and lateral watersupply to the river. The peak runoff, lowered by the crest subsidence, is raised continuously or by impulses to higher values again. So an interaction takes place. It is possible that in some cases the peak runoff of a flood wave fluctuates around a certain value when passing the river. In this case we remain in a more or less fixed point on the abscissa, but it is necessary to apply a decreasing peak curvature. This is due to the wave shape becoming more obtused along the section because of the increasing watervolume. Finally this results in a lower subsidence value than the graphical representation would indicate.

Now return to figure 8. The four factors, mentioned in the foregoing will transform the theoretical trend *c-d-E-j-k* into a gradual transition like *c-l-j-k*.

## 7. APPLICATION TO FLOODS ON THE RIVER MEUSE IN THE NETHERLANDS

Next we will consider an actual situation, related to the river Meuse in the Netherlands. Considered is the river reach Linne-Ravenstein, which has a length of 113 km. Here the next data are valid:

Cross section:

$$\begin{aligned} B_s &= 100 \text{ m} \\ h &= 8.60 \text{ m} \\ p &= 1; (1+p)^2 = 4 \\ n &= 5 \\ m &= 14; (1+n+m)^2 = 400 \end{aligned}$$

Longitudinal section:

$$\begin{aligned} S_b &= 1,0 \cdot 10^{-4} \\ \Delta x &= 113 \text{ km} \end{aligned}$$

Bottom roughness:

$$K = 38.5 \text{ m}^{1/3}/\text{sec}$$

Wave shape:

$$R = 33 \cdot 10^{-12} \text{ sec}^{-2}$$

These factors produce:

$$\begin{aligned} Q_f &= 1400 \text{ m}^3/\text{sec} \\ \Delta Q_f &= 18.8 \text{ m}^3/\text{sec} \end{aligned}$$

From these data figure 11 was composed. Next a series of points, related to the data of the flood waves, that occurred in the period 1941-1960 was plotted. The points give the relative subsidence values derived from the differences between the peak runoff values at respectively the upstream and the downstream ends of the reach considered, and further augmented by the total lateral supply along this reach. They appear to fit reasonably the model elaborated before. All the points are situated in the transition zone between the low water and the high water bed.

The points for lower discharges are omitted, because then backwater effects occur by operating the weirs, leading to a different situation. Higher floods than the plotted ones did not occur in the period considered.

The scattering of the plots is a consequence of partly unknown variations of the peak curvatures and of uncertainties in the runoff values of the tributaries and the in- and outflow at the upstream and downstream ends at the river section. The subsidence values are the calculated differences between two relatively high discharge values. This has an unfavourable effect on the accuracy.

Since some years however, the methods of observation are being improved, whereas the number of observations is expanded. So we hope to be able in the near future to obtain results between narrower limits.

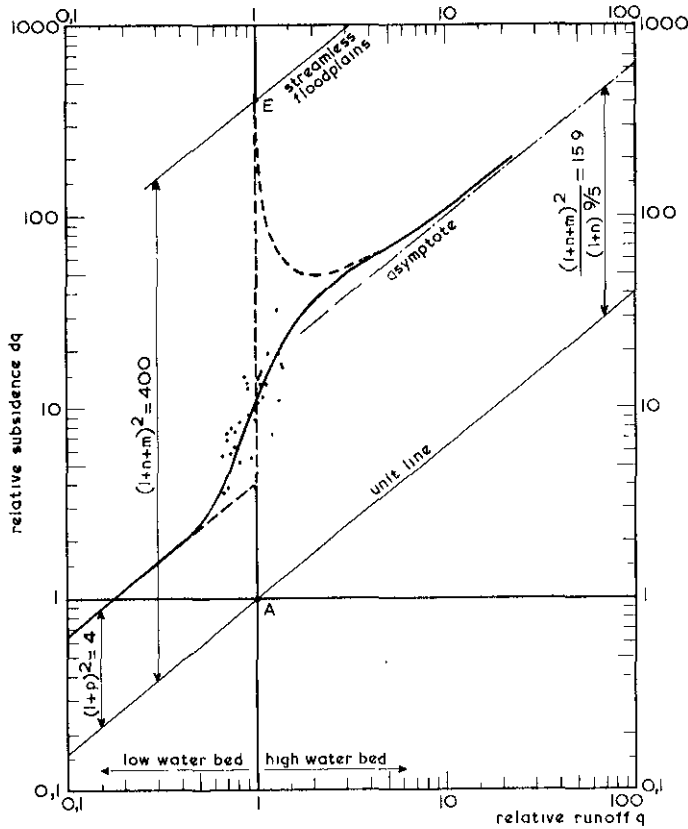


FIGURE 11. River Meuse section Linne-Ravenstein 113 km

ratios applied:

low water bed  $p = 1,0$

high water bed  $n = 5,0$

$m = 14,0$

base data:  $Q_f = 1400 \text{ m}^3/\text{sec}$

$\Delta Q_f = 18,8 \text{ m}^3/\text{sec}$

So far however, the concurrence between the theoretical scheme and the actual data is not unsatisfactory.

## REFERENCES

1. FORCHHEIMER, P., *Hydraulik* (3rd edition). Teubner, Leipzig and Berlin, 1930.
2. VAN DER MADE, J.W., (1966): Flood prevention by enlargement of flood wave subsidence IASH, publication no 71, Symposium of Garda, 1966.